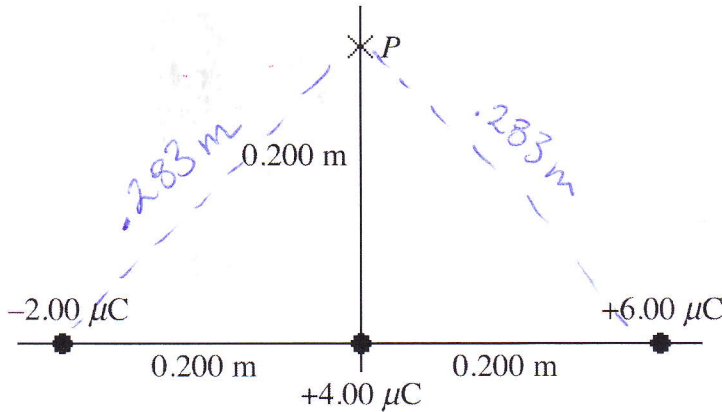


**Final Exam** (80 points)

**Multiple Choice:** Select the best answer for each of the following questions. Write your answer as an English letter to the left of each problem.

- (2pts) 1. Three point charges of  $-2.00 \mu\text{C}$ ,  $+4.00 \mu\text{C}$ , and  $+6.00 \mu\text{C}$  are placed along the  $x$ -axis as shown in the figure. What is the electrical potential at point  $P$  (relative to infinity) due to these charges?



Pt. charge  $V = \frac{Q}{4\pi\epsilon_0 r}$

$$V_{\text{tot}} = \sum V_i$$

$$= \frac{-2\mu\text{C}}{4\pi\epsilon_0(0.283\text{m})} + \frac{4\mu\text{C}}{4\pi\epsilon_0(0.2\text{m})} + \frac{6\mu\text{C}}{4\pi\epsilon_0(0.283\text{m})} = \boxed{307 \text{ kV}}$$

- (a)  $-307 \text{ kV}$     (b)   $+307 \text{ kV}$     (c)  $-154 \text{ kV}$     (d)  $+154 \text{ kV}$     (e)  $0 \text{ kV}$

- (2pts) 2. A charged capacitor stores energy  $U$ . Without connecting this capacitor to anything, dielectric having dielectric constant  $\kappa$  is now inserted between the plates of the capacitor, completely filling the space between them. How much energy does the capacitor now store?

- (a)  $2\kappa U$     (b)  $\kappa U$     (c)  $U$     (d)   $U/\kappa$     (e)  $U/(2\kappa)$

$Q \uparrow$   
 $-Q \downarrow$  Cap. not connected to anything  $\therefore Q$  const.

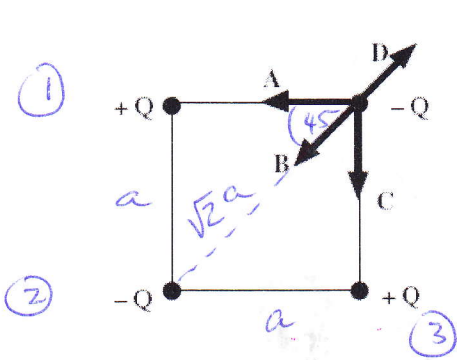
$$U_0 = \frac{Q^2}{2C_0}$$

when insert dielectric  $C = \kappa C_0$

$$\therefore U = \frac{Q^2}{2C} = \frac{Q^2}{2\kappa C_0} = \frac{1}{\kappa} \frac{Q^2}{2C_0} = \boxed{\frac{U_0}{\kappa}}$$

4 pts

(2pts) 3. Four point charges, each of the same magnitude, but with varying signs, are arranged at the corners of a square as shown. Which of the arrows labeled A, B, C, and D gives the correct direction of the net force that acts on the charge at the upper right corner?



- (a) A
- (b) B
- (c) C
- (d) D
- (e) The net force is zero.

$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}$  repulsive along diagonal  
 $F_2 = \frac{Q^2}{4\pi\epsilon_0 (\sqrt{2}a)^2} = \frac{Q^2}{4\pi\epsilon_0 2a^2}$   
 attractive along sides, but only components along diagonal survive

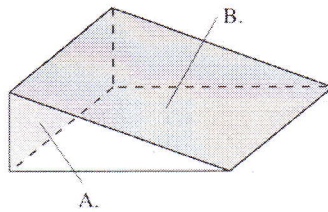
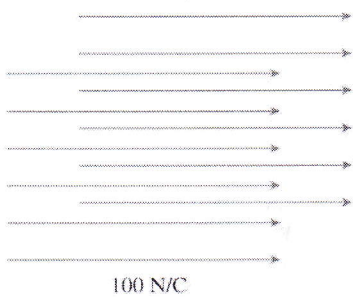
$$F_1 = \frac{Q^2}{4\pi\epsilon_0 a^2} \cos 45 = \frac{Q^2}{4\pi\epsilon_0 a^2 \sqrt{2}} = F_3$$

$$\therefore F_{net} = \frac{2Q^2}{4\pi\epsilon_0 a^2} \left(\frac{1}{\sqrt{2}}\right) - \frac{Q^2}{4\pi\epsilon_0 a^2} \left(\frac{1}{2}\right)$$

towards ②                      away from ②

$$= \frac{Q^2}{4\pi\epsilon_0 a^2} \left(\sqrt{2} - \frac{1}{2}\right) \dots \text{towards } \textcircled{2}$$

(2pts) 4. Which surface, A or B, has the largest electrical flux? A is the vertical rectangular surface and B is the broad sloped surface.

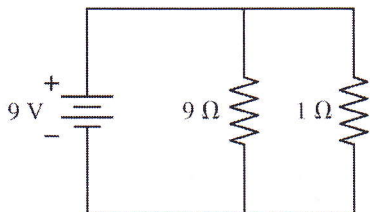


same no. of  $\vec{E}$ -field lines pass through A & B.

$\therefore$  same flux through two surfaces.

- (a) A
- (b) B
- (c) The electrical fluxes are equal.
- (d) More information is needed.

(2pts) 5. Which resistor dissipates more power?



- (a) 1 Ω
- (b) 9 Ω
- (c) They dissipate the same power.
- (d) No power is dissipated.

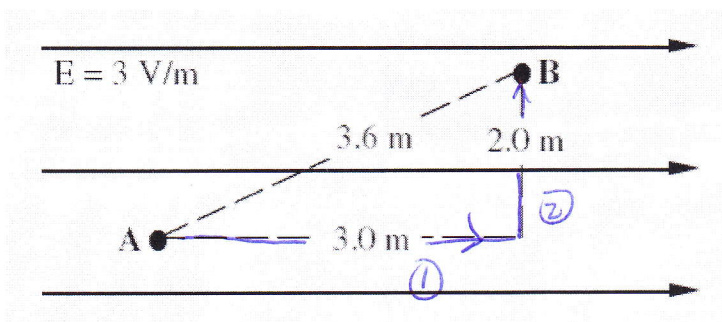
$$P = \frac{V^2}{R}$$

$$P_9 = \frac{(9V)^2}{9\Omega} = 9W$$

$$P_1 = \frac{(9V)^2}{1\Omega} = 81W$$

$$P_1 > P_9$$

(2pts) 6. A +1.0 μC point charge is moved from point A to B in the uniform electric field as shown. What happens to potential energy of the point charge as a result of this move?



- (a) It increases by  $6.0 \times 10^{-6}$  J
- (b) It decreases by  $6.0 \times 10^{-6}$  J
- (c) It decreases by  $9.0 \times 10^{-6}$  J
- (d) It increases by  $10.8 \times 10^{-6}$  J
- (e) It decreases by  $10.8 \times 10^{-6}$  J

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$= \underbrace{-\int \vec{E} \cdot (dx \hat{i})}_{\textcircled{1}} - \underbrace{\int \vec{E} \cdot (dy \hat{j})}_{\textcircled{2}}$$

$$\vec{E} = 3 \frac{V}{m} \hat{i}$$

$$\Delta V = -\int_0^3 3 \frac{V}{m} dx = \left(-3 \frac{V}{m}\right)(3m)$$

$$= -9V$$

$$\Delta U = q\Delta V = (1\mu C)(-9V)$$

$$\boxed{-9 \times 10^{-6} \text{ J}}$$

decrease.

4pts

(2pts) 7. A solid nonconducting sphere of radius  $R$  carries a uniform charge density throughout its volume. At a radial distance  $r_1 = R/4$  from the centre, the electric field has a magnitude  $E_0$ . What is the magnitude of the electric field at a radial distance  $r_2 = 2R$ ?

- (a)  $E_0/4$       (b) zero      (c)  $E_0/2$       (d)  $E_0$       (e)  $2E_0$

Gauss's Law

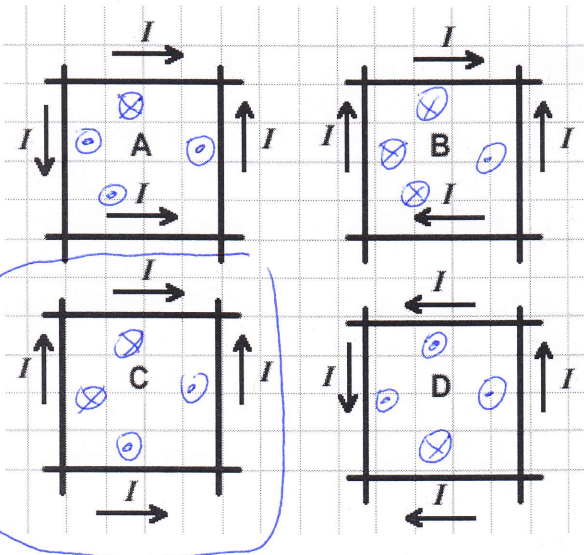
$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$q_{in} \text{ for } r_1 = \frac{R}{4} \text{ is } q_1 = \rho \frac{4}{3}\pi \left(\frac{R}{4}\right)^3 = \frac{Q}{4^3}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \Rightarrow E_0 4\pi \left(\frac{R^2}{4}\right)^2 = \frac{Q}{4^3 \epsilon_0} \Rightarrow E_0 = \frac{Q/4}{4\pi \epsilon_0 R^2}$$

when  $r_2 = 2R$   $q_{in}$  is  $Q$ .  $\therefore E_2 = \frac{Q}{4\pi \epsilon_0 (2R)^2} = \frac{Q/4}{4\pi \epsilon_0 R^2} = E_0$  ← same

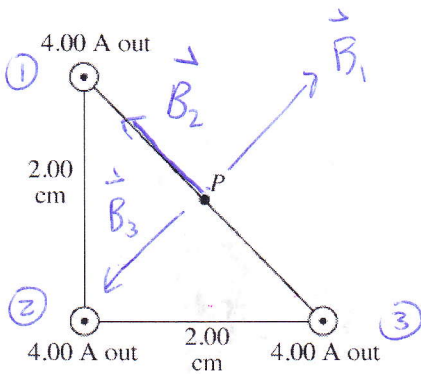
(2pts) 8. The figure shows four different sets of insulated wires that cross each other at right angles without actually making electrical contact. The magnitude of the current is the same in all the wires, and the directions of the current flow are as indicated. For which, if any, configuration will the magnetic field at the centre of the square formed by the wires be equal to zero?



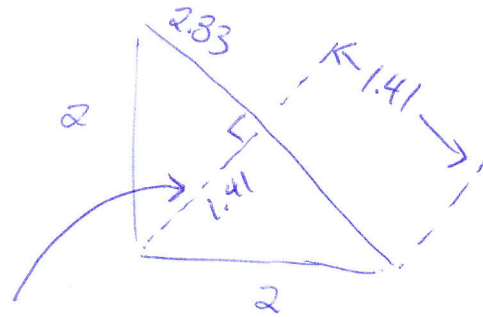
use RHR  
require currents across from one another to produce  $\vec{B}$  that cancels inside square loop.

- (a) A  
(b) B  
(c) C  
(d) D  
(e) The field is not equal to zero in any of these cases.

- (2pts) 9. Three very long, straight, parallel wires each carry currents of 4.00 A directed out of the page as shown in the figure. The wires pass through the vertices of a right isosceles triangle of side 2.00 cm. What is the magnitude of the magnetic field at point P at the midpoint of the hypotenuse of the triangle?



$\vec{B}_1$  &  $\vec{B}_3$  cancel  
only consider  $\vec{B}_2$ .

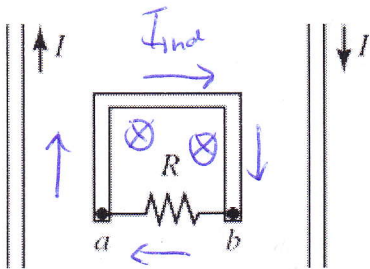


$\sqrt{2^2 - 1.41^2} =$

- (a)  $1.77 \times 10^{-6}$  T
- (b)  $4.42 \times 10^{-6}$  T
- (c)  $1.77 \times 10^{-5}$  T
- (d)  $5.66 \times 10^{-5}$  T
- (e)  $1.26 \times 10^{-4}$  T

$$B_2 = \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 (4 \text{ A})}{2\pi (0.0141 \text{ m})} = \boxed{5.67 \times 10^{-5} \text{ T}}$$

- (2pts) 10. In the figure, two parallel wires carry currents of magnitude  $I$  in opposite directions. A rectangular loop is midway between the wires. The current  $I$  is decreasing with time. The induced current through resistor  $R$  is:



Lenz' law

since  $I$  decrease,  $\Phi_m$  decreasing.

To oppose change  $\vec{B}_{ind}$  must be into page

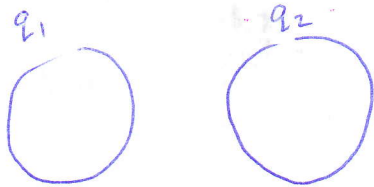
$\therefore I_{ind}$  from b to a through R

- (a) from a to b.
- (b) from b to a
- (c) There is no induced current through the resistor.

**Free Response:** Write out complete answers to the following questions. Show your work since it allows us to be generous with partial credit.

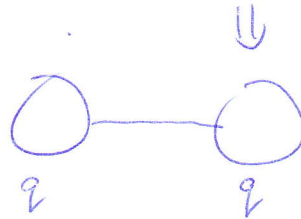
- (10pts) 11. a) Two identical positively charged conducting spheres are fixed in space. The spheres are 32.6 cm apart (centre to centre) and repel each other with an electrostatic force  $F_1 = 0.0615$  N. Then, a thin conducting wire connects the spheres, redistributing the charge on each sphere. When the wire is removed the spheres still repel but with a force of  $F_2 = 0.115$  N. Use this information to find the initial charge on each sphere. Assume that initially  $q_1 < q_2$ . (6 marks)

- b) Initially, a conducting sphere of radius  $R$  has charge  $Q$ . Nearby, there is a neutral conducting of radius  $R/3$ . The two spheres are connected by a thin conducting wire. After connecting the wire, what is the charge on the smaller sphere? Express your answer in terms of  $Q$ .



when connect w/ conducting wire  
two spheres will have same charge since  
they are identical.

$$F_1 = \frac{q_1 q_2}{4\pi\epsilon_0 d^2}$$



$$F_2 = \frac{q^2}{4\pi\epsilon_0 d^2}$$

$$F_2 = 0.115 \text{ N}$$

$$d = 0.326 \text{ m}$$

$$\therefore q = 1.17 \mu\text{C}$$

conservation of charge  
requires

$$q_1 + q_2 = 2q \Rightarrow q_1 = 2q - q_2$$

$$\therefore F_1 = \frac{(2q - q_2)q_2}{4\pi\epsilon_0 d^2} \Rightarrow 4\pi\epsilon_0 d^2 F_1 = 2q q_2 - q_2^2 \Rightarrow q_2^2 - 2q q_2 + 4\pi\epsilon_0 d^2 F_1 = 0$$

$$\therefore q_2^2 - 2.33 \times 10^{-6} q_2 + 7.27 \times 10^{-13} = 0$$

quadratic formula:  $q_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$q_2 = \begin{cases} 1.96 \mu\text{C} \\ 0.371 \mu\text{C} \end{cases}$$

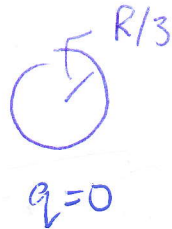
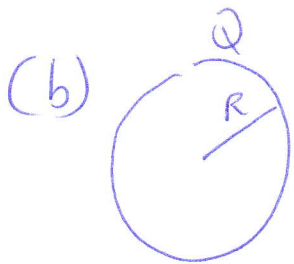
take  $q_2 = 1.96 \mu\text{C}$  since  $q_2 > q_1$

$$q_1 = 2q - q_2 = 0.37 \mu\text{C}$$

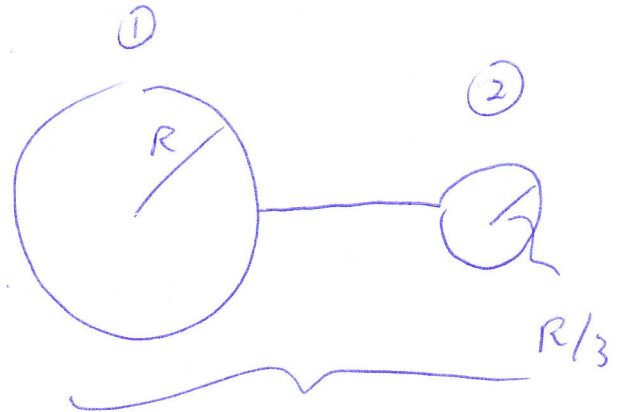
10 pts

check

$$F_1 = \frac{q_1 q_2}{4\pi\epsilon_0 d^2} = 0.0614 \text{ N } \checkmark$$



$\Rightarrow$



one single conductor  
 $\therefore$  potential same everywhere.

After connected by wire,  
charge  $q$  from ① move to ②

$$\therefore Q_1 = Q - q \quad Q_2 = q$$

$$V_1 = \frac{Q_1}{4\pi\epsilon_0 R} = \frac{Q - q}{4\pi\epsilon_0 R}$$

$$V_2 = \frac{Q_2}{4\pi\epsilon_0 R/3} = \frac{3q}{4\pi\epsilon_0 R}$$

if  $V_1 = V_2$ , then

$$\frac{Q - q}{4\pi\epsilon_0 R} = \frac{3q}{4\pi\epsilon_0 R}$$

$$\therefore Q - q = 3q$$

$$\therefore Q = 4q \Rightarrow$$

$$q = \frac{Q}{4}$$

charge of smaller sphere.

Note larger sphere has charge

$$Q_1 = Q - q = \frac{3Q}{4}$$

(10pts) 12. a) You are in the lab and need a 90 pF capacitor to properly set the time constant of an RC circuit. Unfortunately, the lab is not equipped with 90 pF capacitors. Your TA gives you one 80 pF  $\pm 10\%$  capacitor, one 80 pF  $\pm 15\%$  capacitor, and one 50 pF  $\pm 20\%$  capacitor. Your TA tells you to combine these capacitors make the necessary 90 pF. How should you combine these capacitors? Draw a circuit diagram and label which capacitors are 80 pF and which is 50 pF. (5 marks)

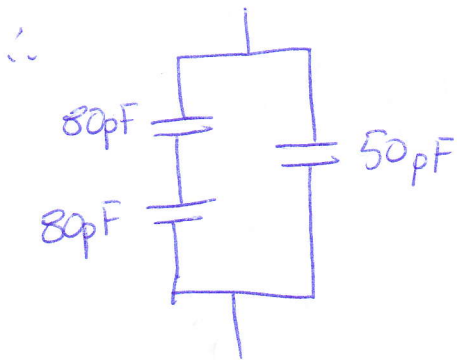
b) What is the uncertainty in the capacitance of the 90 pF circuit that you constructed in part a)? Give your answer in pF. (5 marks)

(a) Put two 80 pF caps in series

$$\begin{array}{c} \text{---} \parallel \text{---} \\ 80\text{pF} \quad 80\text{pF} \end{array} \Rightarrow \begin{array}{c} \text{---} \parallel \text{---} \\ C_{eq1} \end{array} \quad \frac{1}{C_{eq1}} = \frac{1}{80\text{pF}} + \frac{1}{80\text{pF}} \Rightarrow C_{eq1} = 40\text{pF}$$

Put 40 pF in parallel to 50 pF.

$$\begin{array}{c} \square \\ \parallel \\ 50\text{pF} \quad C_{eq1} \end{array} \Rightarrow \begin{array}{c} \text{---} \parallel \text{---} \\ C_{eq} \end{array} \quad C_{eq} = 50\text{pF} + C_{eq1} = 90\text{pF} \checkmark$$



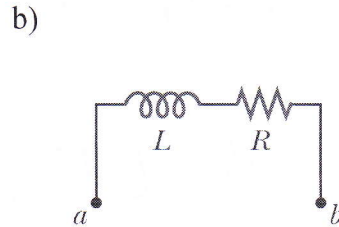
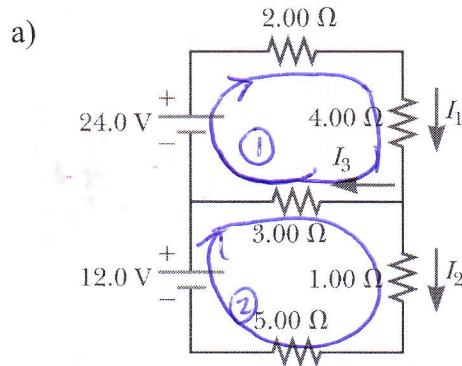
gives equivalent capacitance of 90 pF.



$$\begin{aligned}
(b) \quad C_{eq} &= \left( \frac{1}{80\text{pF}} + \frac{1}{80\text{pF}} \right)^{-1} + 50\text{pF} \\
&= \left( \frac{1}{80\text{pF} \pm 10\%} + \frac{1}{80\text{pF} \pm 15\%} \right)^{-1} + 50\text{pF} \pm 20\% \\
&= \left( 0.0125 \pm 10\% + 0.0125 \pm 15\% \right)^{-1} + (50\text{pF} \pm 10\text{pF}) \\
&= \left( 0.0125 \pm 0.00125 + 0.0125 \pm 0.001875 \right)^{-1} + (50\text{pF} \pm 10\text{pF}) \\
&= \left( 0.025 \pm 0.003125 \right)^{-1} + (50\text{pF} \pm 10\text{pF}) \\
&= \left( 0.025 \pm 12.5\% \right)^{-1} + (50\text{pF} \pm 10\text{pF}) \\
&= \left( 40\text{pF} \pm 12.5\% \right) + (50\text{pF} \pm 10\text{pF}) \\
&= \left( 40\text{pF} \pm 5\text{pF} \right) + (50\text{pF} \pm 10\text{pF}) \\
&= \boxed{90\text{pF} \pm 15\text{pF}} \quad (16.7\%)
\end{aligned}$$

(10pts) 13. a) Find the currents in the circuit shown in figure a). Are they shown in the correct direction? (Do **not** change the directions of the currents.) (5 marks)

b) When the current in the portion of the circuit shown in figure b) is 2.00 A and increases at a rate of 0.500 A/s, the measured voltage is  $\Delta V_{ab} = 9.00$  V. When the current is 2.00 A and decreases at the a rate of 0.500 A/s, the measured voltage is  $\Delta V_{ab} = 5.00$  V. Calculate the values of  $L$  and  $R$ . (5 marks)



(a) From ①

$$\textcircled{i} \quad 24 - 2I_1 - 4I_1 - 3I_3 = 0 \Rightarrow 24 - 6I_1 - 3I_3 = 0$$

From ②

$$\textcircled{ii} \quad 12 + 3I_3 - I_2 - 5I_2 = 0 \Rightarrow 12 - 6I_2 + 3I_3 = 0$$

Jen rule

$$\textcircled{iii} \quad I_1 = I_2 + I_3$$

sub ③ into ①  $24 - 6(I_2 + I_3) - 3I_3 = 0$

$$\textcircled{iv} \quad 24 - 6I_2 - 9I_3 = 0$$

$$\textcircled{iv} - \textcircled{ii} \Rightarrow 12 - 12I_3 = 0 \quad \therefore \boxed{I_3 = 1.0 \text{ A}}$$

$$\therefore 12 + 3(1 \text{ A}) = 6I_2 \quad \therefore I_2 = \frac{15}{6} \text{ A} = \frac{5}{2} \text{ A} = \boxed{2.50 \text{ A} = I_2}$$

$$\therefore I_1 = I_2 + I_3 = \boxed{3.50 \text{ A} = I_1}$$

All currents  
shown in  
correct  
dir'n.

check using outside loop.

$$\underline{\underline{24 - 2I_1 - 4I_1 - I_2 - 5I_2 + 12 = 0?}}$$

$$24 - 6(3.5) - 6(2.5) + 12 = 0 \checkmark$$

(b)

$$V_a - V_L - V_R = V_b$$

$$V_R = IR$$

$$V_L = L \frac{dI}{dt}$$

$$\Delta V_{ab} = V_a - V_b = L \frac{dI}{dt} + IR$$

$$\therefore 9V = L \left( +0.5 \frac{A}{s} \right) + (2A)R \quad \textcircled{1}$$

↑  
increasing

$$5V = L \left( -0.5 \frac{A}{s} \right) + (2A)R \quad \textcircled{2}$$

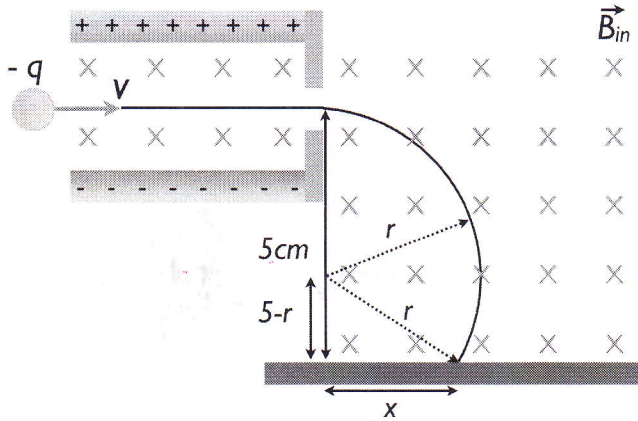
↑  
decreasing

$$\textcircled{1} + \textcircled{2} \quad 14V = (4A)R \quad \Rightarrow \quad R = \frac{14V}{4A} = 3.5 \Omega$$

$$\therefore L \left( 0.5 \frac{A}{s} \right) = (2A)R - 5V$$

$$L = \frac{2(3.5) - 5}{0.5} = 4.0 \text{ H} = L$$

- (10pts) 14. Consider the mass spectrometer shown below. The magnitude of the electric field between the plates of the capacitor is  $E = 1000 \text{ V/m}$ . A uniform magnetic field is directed into the page and has a magnitude of  $0.1 \text{ T}$ . Calculate the position  $x$  at which a particle of mass  $m = 7.3 \times 10^{-26} \text{ kg}$  and charge  $q = 1.6 \times 10^{-19} \text{ C}$  will hit the plate.



when inside capacitor  
FBD is:

$$\vec{F}_E = q\vec{E}$$

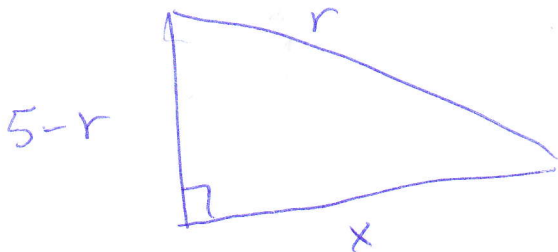
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

charges only make it through if  
 $F_E = F_B$

$$qE = qvB \Rightarrow \boxed{v = \frac{E}{B}}$$

When outside capacitor only force is magnetic force which causes circular motion.

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB} = \frac{mE}{qB^2} = 4.56 \text{ cm}$$



$$(5-r)^2 + x^2 = r^2$$

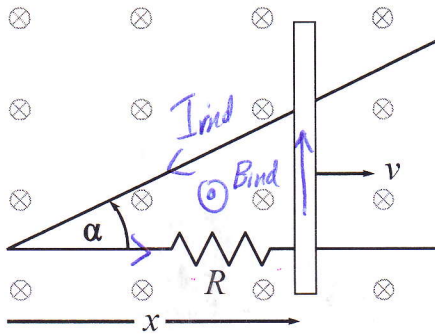
$$25 - 10r + r^2 + x^2 = r^2$$

$$x^2 = 10r - 25$$

$$x = \sqrt{10r - 25}$$

$$\Rightarrow \boxed{x = 4.54 \text{ cm}}$$

- (10pts) 15. A triangular circuit of total resistance  $R$  is made from a conducting bar that can slide along a conducting track. A uniform magnetic field of magnitude  $B$  is directed into the page as shown. If the bar is pulled to the right with speed  $v$ , find an expression for the induced current in terms of  $B$ ,  $v$ ,  $x$ ,  $\alpha$ , and  $R$ . Is the induced current directed clockwise or counterclockwise?

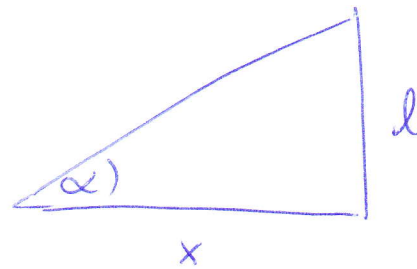


Flux through loop increasing  
(area increases as pull bar).  
To oppose change  $\vec{B}_{ind}$  out of page

$\therefore I_{ind}$  is counterclockwise.

$$I_{ind} = \frac{|\mathcal{E}|}{R} \quad |\mathcal{E}| = \left| \frac{d\Phi_m}{dt} \right|$$

$$\Phi_m = \vec{B} \cdot \vec{A} = BA \cos 0 = BA$$



$A = \frac{1}{2} x l$  but  $l$  changes as  $x$  changes.

$$\tan \alpha = \frac{l}{x} \quad \therefore l = x \tan \alpha$$

$$\therefore A = \frac{1}{2} x (x \tan \alpha) = \frac{1}{2} x^2 \tan \alpha$$

$$\therefore \Phi_m = \frac{B}{2} x^2 \tan \alpha$$

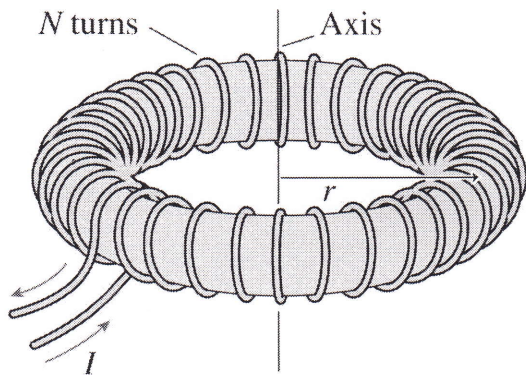
chain rule!

$$\frac{d\Phi_m}{dt} = \frac{B}{2} \tan \alpha \left( \frac{d}{dt} x^2 \right) = \frac{B}{2} \tan \alpha \cdot 2x \frac{dx}{dt}$$

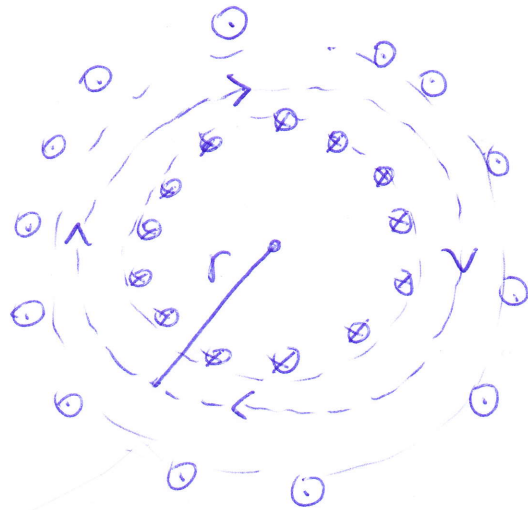
$$\therefore \mathcal{E} = \frac{d\Phi_m}{dt} = Bxv \tan \alpha$$

$$\therefore I_{ind} = \mathcal{E}/R = \frac{Bxv}{R} \tan \alpha$$

- (10<sup>pts</sup>) 16. The toroid is a coil of wire wrapped around a doughnut-shaped ring made of a nonconducting material. Find an expression for the magnetic field strength at a point inside the coil of wire and a distance  $r$  from the central axis.



Bottom View



Like a solenoid  $B \approx 0$   
outside coil.

Use Ampère's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

choose path inside coil

Expect  $B$  to be const. along path provided path is circular with radius  $r$ . Also, expect  $\vec{B}$  to be circular inside toroid.

$$\therefore \oint \vec{B} \cdot d\vec{s} = \oint B ds = B \int ds = B 2\pi r$$

$I_{\text{through}} = NI$  (only the inside currents going into the page contribute to  $I_{\text{through}}$ )

$$\therefore B 2\pi r = \mu_0 NI \quad \Rightarrow \quad \boxed{B = \frac{\mu_0 NI}{2\pi r}}$$